

LINEAR STABILITY OF LIQUID FILMS WITH INTERFACIAL PHASE CHANGE

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Abstract—The influence of the interfacial phase change on the stability of liquid film flow over a plane is studied. It is significant in the interfacial mass balance equation only. Evaporation has a destabilizing influence and condensation a stabilizing one. The thickness variation caused by the phase change is taken into account. Its effect on the stability is the opposite of the phase change effect.

NOMENCLATURE

<p>A, film thickness; a, dimensionless film thickness $\triangleq A/Y_{Lr}$; Br, Brinkman number $\triangleq \mu_r U_r^2 / \lambda_r \Delta\Theta_r$; C_p, heat capacity; c, velocity of the perturbation $\triangleq c_r + ic_i$; e, $\triangleq +1$ in case of evaporation, $\triangleq -1$ in case of condensation; F, external forces; g, gravitational acceleration; I, enthalpy; Ku, Kutateladze number $\triangleq C_{pv} \Delta\Theta_{Lr}/L$; L, heat of vaporization; l, variation of the thermal conductivity: $\lambda = \lambda_r [1 + l(\Theta - \Theta_r)]$; M, $\triangleq \mu_G/\mu_L$; m, variation of the viscosity: $\mu = \mu_r [1 + m(\Theta - \Theta_r)]$; \dot{m}, mass transfer per unit area and per unit of time $\triangleq \rho(\mathbf{V} - \mathbf{V}_i) \cdot \mathbf{n}$; \mathbf{n}, normal unit vector; P, pressure; Pe, Péclet number $\triangleq U_r Y_r \rho_r C_p / \lambda_r$; Pr_r, Prandtl number $\triangleq \mu_r C_p / \lambda_r$; P^*, modified pressure $\triangleq P_G - g\rho_{Gr} \sin \Omega X + g\rho_{Gr} \cos \Omega [Y - A(X=0, T)]$; p, dimensionless pressure difference $\triangleq (P - P_r)/\Delta P_r$; p_b, dimensionless interfacial pressure difference $\triangleq (P_{Li} - P_{Gi})/\Delta P_{ir}$; p^*, dimensionless modified pressure difference $\triangleq (P^* - P_r^*)/\Delta P_r^*$; q, heat flux; Re, Reynolds number $\triangleq U_r Y_r \rho_r / \mu_r$; \mathcal{T}, stress tensor; t, dimensionless time $\triangleq T/T_r$; \mathbf{t}, tangential unit vector; u, dimensionless longitudinal velocity $\triangleq U/U_r$; u_b, dimensionless interfacial longitudinal velocity difference $\triangleq (U_{Gi} - U_{Li})/\Delta U_r$; \mathbf{V}, velocity vector;</p>	<p>\mathbf{V}_b, interfacial velocity vector: $\mathbf{V}_i \cdot \mathbf{n}_L = [\partial A/\partial T] [1 + (\partial A/\partial X)^2]^{-1/2}$; \mathbf{V}_p, $\triangleq (\mathbf{V}_i \cdot \mathbf{n}_k) \cdot \mathbf{n}_k + (\mathbf{V}_k \cdot \mathbf{t}) \cdot \mathbf{t}$; v, dimensionless transversal velocity $\triangleq V/V_r$; v_b, dimensionless interfacial transversal velocity difference $\triangleq (V_{Gi} - V_{Li})/\Delta V_r$; We, Weber number $\triangleq \sigma_r / \rho_{Lr} U_{Lr}^2 Y_{Lr}$; x, dimensionless longitudinal coordinate $\triangleq X/X_r$; Y, transversal coordinate ($Y = 0$ at the wall); y, dimensionless transversal coordinate: $y_L \triangleq Y/Y_{Lr}$; $y_G \triangleq (Y - Y_{Lr})/Y_{Gr}$;</p> <p>Greek symbols</p> <p>α, dimensionless wave number $\triangleq 2\pi Y_{Lr}/\Lambda$; β, variation of the density: $\rho = \rho_r [1 + \beta(\Theta - \Theta_r)]$; γ, $\triangleq \rho_G/\rho_L$; $\Delta\Theta_r$, temperature difference scale; η, ratio of the length scales $\triangleq Y_{Lr}/X_{Lr}$; Θ, temperature; θ, dimensionless temperature $\triangleq (\Theta - \Theta_r)/\Delta\Theta_r$; Λ, wavelength; λ_r, thermal conductivity; μ, dynamic viscosity; ρ, density; σ, surface tension; \mathcal{V}, viscous stress tensor; Ω, angle of inclination of the plane to the horizontal.</p> <p>Indices</p> <p>G, vapor; i, interface; k, $\triangleq (G, L)$; L, liquid; r, scale; S, saturation; w, wall; 0, first-order; 1, second-order.</p>
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1. INTRODUCTION

THE FLOW of liquid films occurs in some natural phenomena (rainwater flow) as in many industrial plant (evaporators, condensers, nuclear reactors, etc.). The waves which propagate at the film surface increase the interfacial transfers [1]. A way to predict their existence is to study the linear stability of the flow.

The studies of the linear stability of isothermal film flow are numerous [2]. Only some authors dealt with film flows with wall heat flux and interfacial phase change. Kocamustafaogullari [3] uses balance equations averaged over the film thickness. He accurately takes into account the phase change influence but neglects the film thickness variation. Bankoff [4], Marschall and Lee [5] and Lin [6], take into account the phase change in the interfacial momentum balance equation only. A detailed criticism of the equations they use was made by Spindler, Solesio and Delhaye [7]. Finally Unsal and Thomas [8], for a condensation film flow only, accurately take into account the phase change, but make some unjustified simplifications.

The aim of this work is to study the linear stability of a liquid film flowing over an inclined plane with wall heat flux and interfacial phase change, without any imposed vapor flow and particularly to make obvious the phase change influence on the stability.

2. EQUATIONS GOVERNING THE FLOW

2.1. General form

The equations governing the flow are the mass, momentum and energy balance equations in the liquid and vapor phase, and at the interface. The fluid is Newtonian and the variations of its physical properties are considered. The only material property of the interface which is taken into account is the surface tension. No vapor flow is imposed, but the vapor layer set into motion by the film flow is considered.

The mass, momentum and energy equations in each phase read

$$\frac{\partial \rho}{\partial T} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

$$\frac{\partial \rho \mathbf{V}}{\partial T} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) - \rho \mathbf{F} - \nabla \cdot \mathcal{F} = \mathbf{0} \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial T} (\rho I - P) + \nabla \cdot (\rho I \mathbf{V}) \\ - \mathbf{V} \cdot \nabla P + \nabla \cdot \mathbf{q} - \mathcal{V} : \nabla \mathbf{V} = 0. \end{aligned} \quad (3)$$

The interfacial mass, momentum and energy equations read [9]

$$\dot{m}_L + \dot{m}_G = 0 \quad (4)$$

$$\begin{aligned} \dot{m}_L \mathbf{V}_L + \dot{m}_G \mathbf{V}_G - \mathbf{n}_L \cdot \mathcal{F}_L - \mathbf{n}_G \mathcal{F}_G \\ + \nabla_s \sigma - (\nabla_s \cdot \mathbf{n}_k) \sigma \mathbf{n}_k = \mathbf{0} \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{m}_L (I_L + \frac{1}{2} V_L^2 - \mathbf{V}_L \cdot \mathbf{V}_p) + \dot{m}_G (I_G + \frac{1}{2} V_G^2 - \mathbf{V}_G \cdot \mathbf{V}_p) \\ + \mathbf{q}_L \cdot \mathbf{n}_L + \mathbf{q}_G \cdot \mathbf{n}_G - (\mathcal{V}_L \cdot \mathbf{n}_L) \cdot (\mathbf{V}_L - \mathbf{V}_p) \end{aligned}$$

$$- (\mathcal{V}_G \cdot \mathbf{n}_G) \cdot (\mathbf{V}_L - \mathbf{V}_p) - \frac{d_s \sigma}{dt} = 0. \quad (6)$$

The wall conditions are constant temperature (CT) or constant heat flux (CHF) and the no-slip condition

$$\text{CT: } \Theta_L = \Theta_w \quad (7)$$

$$\text{CHF: } -\lambda_L \frac{\partial \Theta}{\partial Y} = q_w$$

$$\mathbf{V}_L = \mathbf{0}. \quad (8)$$

The interfacial boundary conditions are the equality of the liquid and vapor temperature and the no-slip condition:

$$\Theta_L = \Theta_G \quad (9)$$

$$\mathbf{V}_L \cdot \mathbf{t} = \mathbf{V}_G \cdot \mathbf{t}. \quad (10)$$

Finally, the vapor is supposed to be at saturation temperature, and out of the layer set into motion by the film flow, the vapor is at rest:

$$\Theta_G = \Theta_s \quad (11)$$

$$\lim_{Y \rightarrow \alpha} \mathbf{V}_G = \mathbf{0}. \quad (12)$$

2.2. Simplification methodology

To simplify the problem the order of magnitude of each term in the dimensionless equations is classically analyzed. Each variable is referred to a certain scale, so that the dimensionless variable and its derivative have a value close to one.

The first stage of the simplification process is to determine the value of each scale. The following hypotheses are made

- water at saturation under atmospheric pressure is chosen as reference fluid. The study can be extended to other fluids if they support the same simplifications. It is the case for instance of refrigerant R12;
- the ratios μ_G/μ_L and ρ_G/ρ_L of the vapor to liquid viscosities and densities are supposed to be small;
- in the moving vapor layer, viscosity forces balance inertia forces;
- on the other hand, in the liquid film, viscosity forces balance gravity forces, inertia forces being negligible. Finally, the convection heat transfer is neglected: the heat flux essentially allows the phase change to take place.

Considering the dimensionless equations, these hypotheses lead to the determination of all the scales [10]. The results are given in Appendix 1. All the scales are expressed in terms of two of them, the liquid temperature difference scale $\Delta \Theta_{Lr}$, and one of the liquid length scales, Y_{Lr} or X_{Lr} , or in terms of two dimensionless numbers, the Reynolds number Re and the Kutateladze number Ku , which represents the ratio of the convective heat to the heat of vaporization.

As a consequence of neglecting the convection in the liquid film, the following limitation has to be introduced

$$Ku \ll 1$$

or

$$\Delta\theta_{Lr} \ll L/C_{PrL}$$

The convection is negligible if the wall to interface temperature difference (or the wall heat flux) is not too large. For saturated water under atmospheric pressure ($L = 2.3 \times 10^6 \text{ J kg}^{-1}$, $C_p = 4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$), a Kutateladze number limitation of 10^{-2} corresponds to a $\Delta\Theta_{Lr}$ limitation of 5.5 K.

The main result is the value of the liquid length scales ratio,

$$\eta_L = \frac{Ku}{Pr_L Re_L} \quad (14)$$

which on account of condition (13) is much less than one (for water at 100°C , $Pr_L = 1.7$). The significance of equation (14) is that the conduction heat balances the vaporization heat (Ku is the convection to vaporization ratio; $\eta_L Pr_L Re_L$ is the convection to conduction ratio). As the transversal to longitudinal liquid velocities ratio is also equal to $\eta_L (V_{Lr}/U_{Lr} = \eta_L)$, equation (14) gives an evaluation of the non-parallelism of the flow, i.e. of the transversal velocity scale (for the isothermal base flow, $V_L = 0$).

It is now possible to deal with the second stage of the simplification process, which is the calculation of the order of magnitude of the different terms in the dimensionless equations. Details of this work can be found in the report by Spindler [10] for the base flow equations as well as for the perturbed flow equations.

The Kutateladze number is chosen having a value close to 10^{-2} and the Reynolds number to vary between 1 and 10^3 (the limiting value of laminar flow as given by Fulford [11] is $Re = 1200$).

Terms with a value close to one (first-order, left-hand side of the equations below), as well as terms with a very close to 10^{-2} (second-order, right-hand side of the equations) are retained. The lower are discarded.

3. BASE FLOW

3.1. Simplified equations

The base flow (steady-state flow) equations are obtained by setting all the time derivatives to zero.

Liquid balance equations (index L is omitted)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\beta \Delta\Theta_r \left[\theta \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right] \quad (15)$$

$$1 + \frac{\partial^2 u}{\partial y^2} = \eta Re \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) - \beta \Delta\Theta_r \theta + \gamma_r + \eta \frac{\partial p}{\partial x} - m \Delta\Theta_r \theta \frac{\partial^2 u}{\partial y^2} - m \Delta\Theta_r \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} \quad (16)$$

$$\cotg \Omega + \frac{\partial p}{\partial y} = -\cotg \Omega \beta \Delta\Theta_r \theta$$

$$+ \eta \left[2 \frac{\partial^2 v}{\partial y^2} - \frac{2}{3} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \frac{\partial^2 u}{\partial y^2} \right] \quad (17)$$

$$\frac{\partial^2 \theta}{\partial y^2} = \eta Pe \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) - l \Delta\Theta_r \theta \frac{\partial^2 \theta}{\partial y^2} - l \Delta\Theta_r \left(\frac{\partial \theta}{\partial y} \right)^2 - Br \left(\frac{\partial u}{\partial y} \right)^2 \quad (18)$$

Vapor balance equations (index G is omitted)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (19)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \eta^2 \frac{\partial p^*}{\partial x} - \eta^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0 \quad (20)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial p^*}{\partial y} - \eta^2 \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v}{\partial y^2} = 0 \quad (21)$$

Interfacial balance equations

$$v_L - u_L a_x - v_i = \gamma_r (v_L - u_L a_x) \quad (22)$$

$$p_i - (v_L - u_L a_x)^2 - 2 \frac{\gamma_r}{\eta_L Re_L} \frac{\partial v_L}{\partial y_L} + 2 \frac{\partial v_G}{\partial y_G} + We \gamma_r \frac{\partial^2 a}{\partial x_L^2} = (2\beta_L \Delta\Theta_{Lr} - \gamma_r) (v_L - u_L a_x)^2 - \frac{2}{3} \beta_L \Delta\Theta_{Lr} \frac{\gamma_r}{\eta_L Re_L} \left(u_L \frac{\partial \theta_L}{\partial x_L} + v_L \frac{\partial \theta_L}{\partial y_L} \right) \quad (23)$$

$$\frac{\partial u_L}{\partial y_L} = \eta_L Re_L \left(\frac{\partial u_G}{\partial y_G} + \frac{\eta_L^2}{\gamma_r^2} \frac{\partial v_G}{\partial x_G} \right) \quad (24)$$

$$v_L - u_L a_x + \frac{\partial \theta_L}{\partial y_L} = -\beta_L \Delta\Theta_{Lr} (v_L - u_L a_x) \quad (25)$$

Boundary conditions:

at the wall ($y_L = 0$)

$$u_L = 0 \quad (26)$$

$$v_L = 0 \quad (27)$$

$$\text{CT: } \theta_L - e = 0$$

or

$$\text{CHF: } \frac{\partial \theta_L}{\partial y_L} + e = -l \Delta\Theta_r \theta_L \frac{\partial \theta_L}{\partial y} \quad (28)$$

at the interface, $y_L = a(x_L)$

$$u_i + a_x v_i = 0 \quad (29)$$

$$\theta_L = \frac{\Delta\Theta_{Gr} X_{Lr}}{\Delta\Theta_{Lr} X_{Gr}} x_L \left[1 + \eta_G \left(\frac{\partial p_L}{\partial x_L} + a_x \frac{\partial p_L}{\partial y_L} \right) \right] \quad (30)$$

$$\theta_G - x_G = \frac{Ku^4}{Pr_L^4 Re_L^4 \gamma_r^3 M_r} x_G \frac{\partial p^*}{\partial x_G} - \eta_L x_G \cotg \Omega \quad (31)$$

at infinity

$$\lim_{y_G \rightarrow \infty} u_G = 0 \quad (32)$$

$$\lim_{y_G \rightarrow \infty} v_G = 0 \quad (33)$$

3.2. First-order solution

Taking into account that the liquid pressure difference scale is always very large compared to the

interfacial pressure difference scale, the liquid balance equations (15)–(18) together with the interfacial balance equations (23)–(25) and boundary conditions (26) (28) and (30) can be solved. The first-order solution reads

$$u_0 = a_0 y - \frac{y^2}{2} \tag{34}$$

$$v_0 = -\frac{y^2}{2} \frac{\partial a_0}{\partial x} \tag{35}$$

$$p_0 = \cotg \Omega (a_0 - y) \tag{36}$$

$$\left. \text{CT: } \theta_0 = e \left(1 - \frac{y}{a_0} \right) \right\} \tag{37}$$

or

$$\text{CHF: } \theta_0 = e(a_0 - y).$$

The value of $a_0(x)$ depends on the flow configuration: a flow with a pre-existing film or a condensation film flow.

3.3 Flow with a pre-existing film

The flow with a pre-existing film (Fig. 1) depends on both the thermal wall condition and on the initial thickness, which can be chosen as the transversal length scale Y_{Lr} . The initial dimensionless thickness then reads

$$a_0(0) = 1 \tag{38}$$

and the Reynolds number, in term of the initial thickness

$$Re_L = g \sin \Omega Y_{Lr}^3 / \nu_{Lr}^2 \tag{39}$$

Inserting equations (34), (35) and (37) into equation (25), we obtain with condition (38) the following results:

$$\text{CT: } a_0 = (1 - 4ex)^{1/4} \tag{40}$$

$$\text{CHF: } a_0 = (1 - 3ex)^{1/3}.$$

In case of evaporation ($e \triangleq +1$), we can calculate a dry-out length which is only theoretical, because on one hand the model used is no longer valid near the dry-out point, and on the other hand the waves

propagating at the film surface also contribute to the dry-out process. The theoretical dry-out length has the following value

$$\begin{aligned} \text{CT: } X_T &= \frac{X_{Lr}}{4} = \frac{Y_{Lr}^4 L \rho_{Lr}^2 g \sin \Omega}{4 \Delta \Theta_{Lr} \lambda_{Lr} \mu_{Lr}} \\ &= \frac{Re_L^{4/3} \nu_{Lr}^{2/3} L \mu_{Lr}}{4 (g \sin \Omega)^{1/3} \Delta \Theta_{Lr} \lambda_{Lr}} \end{aligned} \tag{41}$$

$$\text{CHF: } X_F = \frac{X_{Lr}}{3} = \frac{Y_{Lr}^3 L \rho_{Lr}^2 g \sin \Omega}{3 q_w \mu_{Lr}} = \frac{Re_L L \mu_{Lr}}{3 q_w}.$$

For a vertical flow of saturated water at 100°C ($\rho_{Lr} = 9.6 \times 10^2 \text{ kg m}^{-3}$; $L = 2.3 \times 10^6 \text{ J kg}^{-1}$; $\lambda_{Lr} = 6.8 \times 10^{-1} \text{ W m}^{-1} \text{ K}^{-1}$; $\mu_{Lr} = 2.8 \times 10^{-4} \text{ kg m}^{-1} \text{ s}^{-1}$), with $\Delta \Theta_{Lr} = 5.5 \text{ K}$ ($Ku = 10^{-2}$) or $q_w = 4 \times 10^4 \text{ W m}^{-2}$ ($Ku = 1.0 \times 10^{-2}$ for $Re_L = 93$), we obtain

$$\begin{aligned} X_T &= 0.16 \text{ m} \\ X_F &= 0.27 \text{ m} \end{aligned} \quad \text{for } Re_L = 50$$

and

$$\begin{aligned} X_T &= 8.8 \text{ m} \\ X_F &= 5.4 \text{ m} \end{aligned} \quad \text{for } Re_L = 1000.$$

3.4 Condensation film flow

A condensation film flow (Fig. 2) depends only on the wall thermal condition. The initial thickness condition reads

$$a_0(0) = 0 \tag{42}$$

and leads to the following result:

$$\text{CT: } a_0(x) = (4x)^{1/4} \tag{43}$$

which has been well known since the work of Nüsselt [12]

$$\text{CHF: } a_0(x) = (3x)^{1/3} \tag{44}$$

which was recently obtained by Fujii, Uehara and Oda [13].

The dimensional form of equations (43) and (44):

$$\text{CT: } A_0(X) = \left[\frac{4 \Delta \Theta_{Lr} \lambda_{Lr} \mu_{Lr} X}{L \rho_{Lr}^2 g \sin \Omega} \right]^{1/4} \tag{45}$$

$$\text{CHF: } A_0(X) = \left[\frac{-3 q_w \mu_{Lr} X}{L \rho_{Lr}^2 g \sin \Omega} \right]^{1/3}$$

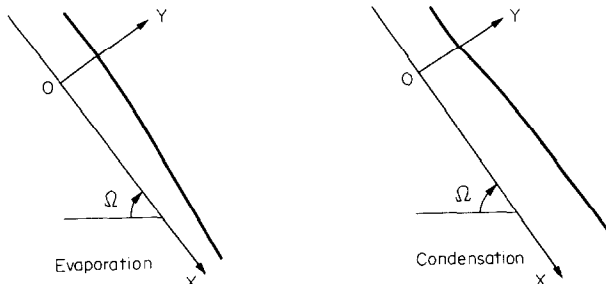


FIG. 1. Flow with a pre-existing film.

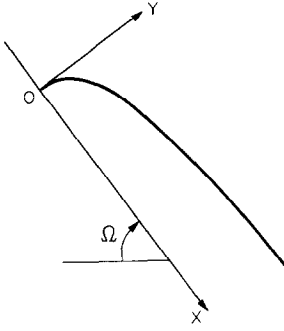


FIG. 2. Condensation film flow.

are expressed in term of the abscissa X , without any requirement of the length scale value.

Consequently, we choose the current abscissa X as the longitudinal length scale X_{Lr} , as for classical boundary layers. The Reynolds number then reads, in term of the current abscissa,

$$\text{CT: } Re_L = [g \sin \Omega \Delta \Theta_{Lr}^3 X^3 \lambda_{Lr}^3 \rho_{Lr}^2 / L^3 \mu_{Lr}^5]^{1/4} \quad (46)$$

$$\text{CHF: } Re_L = eq_w X / L \mu_{Lr}.$$

Nevertheless, the model used is not of course valid near the initial point.

3.5. Second-order solution

The complete second-order solution of the liquid equations can be found in the report by Spindler [10]. Only the dimensionless film thickness (CT) is presented here. It reads

$$a(x_L) = a_0(x_L) + Kua_1(x_L) + O(Ku^2) \quad (47)$$

where $a_0(x_L)$ is given by equations (40), (43) or (44) and $a_1(x_L)$ reads

$$a_1(x_L) = -e[a_0(x_L) - a_0(0)] \left\{ \frac{9}{160Pr_L} - \frac{11}{160} + \frac{\Delta \Theta_{Lr}}{16Ku} \right. \\ \left. \times [\beta_L(4e+3) - 3m_L + 2l_L] - \frac{\gamma_r e}{4Ku} + f(x_L) \right\} \quad (48)$$

with

$$\begin{cases} a_0^4(x) \frac{df}{dx_L} - 3ef(x_L) = \left[\frac{e}{Pr_L} - \frac{a_0^4(x_L)}{2Pr_L} \frac{\partial}{\partial x_L} \right] \\ \quad \times \left(\frac{\partial u_{G0}}{\partial y_G} + \eta_G^2 \frac{\partial v_{G0}}{\partial x_G} \right) \\ f(0) = 0. \end{cases} \quad (49)$$

Equation (48) agrees with the following result of Unsal [14] for a condensation film flow ($e \triangleq -1$; $a_0(0) = 0$), in which neither the liquid property variations, nor the vapor influence are taken into account

$$a_1(x_L) = a_0(x_L) \left(\frac{9}{160Pr_L} - \frac{11}{160} \right). \quad (50)$$

Vapor flow terms appear in the second-order solution of the liquid flow. These terms (among others

$\partial u_{G0}/\partial y_G$) can be calculated only by resolving the complete system of equations governing the vapor flow [equations (19)–(21), (22), (29), (32) and (33)]. This resolution involves heavy calculations except in the case of condensation film flow with constant wall temperature and a low value of η_G (classical boundary-layer type equations).

4. PERTURBED FLOW

4.1. Multiple scale method

The base flow depends on the abscissa x and as a consequence, a perturbation can be from a mathematical point of view only be written under the following form

$$f'(x_L, y_L, t_L) = F(x_L, y_L) \exp(\sigma t_L) \quad (51)$$

(σ is a complex number).

Nevertheless the multiple scale method, developed among others by Bouthier [15] and Eagles and Weissman [16] for quasi-parallel flows ($V \ll U$), assumes a sinusoidal perturbation

$$f'(x_L, y_L, t_L) = F(x_L, y_L) \\ \times \exp \left[\frac{i}{\eta_L} \left(\int^{x_L} \alpha(x) dx - \omega t_L \right) \right] \quad (52)$$

where the wave number (depending on x) appears as the x -derivative of the phase function. Only the spatial stability is considered (α complex, ω real) because precisely the spatial variations of the perturbation characteristics are taken into account.

This method uses the first and the second order equations of the base flow as well as of the perturbed flow. Consequently the main difficulty is the problem of solving first the vapor base flow and then the vapor perturbed flow [10]. As a result the multiple scale method is discarded and replaced by a simpler but less accurate one, the local approximation method, which besides is the first step of the multiple scale method.

4.2. Local approximation method

In the local approximation method the x -dependent flow is replaced by its local value at a fixed abscissa x_0 . The perturbation is then

$$f'(x_L, y_L, t_L) = F_{x_0}(y_L) \exp \left[\frac{i\alpha}{\eta_L} (x_L - ct_L) \right].$$

We no longer consider the x -dependency of the characteristics of one perturbation, but we can calculate the characteristics value of a perturbation initiated at the given abscissa x_0 only.

The dimensionless wave number α is defined with Y_{Lr} as length scale in order to study perturbations the wave length of which has values close to the film thickness.

4.3. Simplified equations

The variables are written as $\bar{f} + f'$, where \bar{f} is the base flow variable and f' its perturbation. Taking into account the base flow equations and discarding all the

nonlinear perturbation terms give the perturbed flow equations.

The order of magnitude of the different terms is examined. In this comparison an x - or t -derivative of a perturbation term corresponds to a multiplication of function F [equation (53)] by $1/\eta_L$ ($\eta_L \ll 1$).

Only the first-order terms are needed in the local approximation method and are presented here. Second-order terms and those corresponding to the vapor flow can be found in Spindler's dissertation [17]. From now on, the index L is always omitted.

Liquid balance equations:

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \quad (54)$$

$$\eta Re \left(\frac{\partial u'}{\partial t} + \bar{u}_0 \frac{\partial u'}{\partial x} + \frac{\partial \bar{u}_0}{\partial y} v' \right) + \eta \frac{\partial p'}{\partial x} - \eta^2 \frac{\partial^2 u'}{\partial x^2} - \frac{\partial^2 u'}{\partial y^2} = 0 \quad (55)$$

$$\eta^2 Re \left(\frac{\partial v'}{\partial t} + \bar{u}_0 \frac{\partial v'}{\partial x} \right) + \frac{\partial p'}{\partial y} - \eta \frac{\partial^2 v'}{\partial y^2} - \eta^3 \frac{\partial^2 v'}{\partial x^2} = 0 \quad (56)$$

$$\eta Pe \left(\frac{\partial \theta'}{\partial t} + \bar{u}_0 \frac{\partial \theta'}{\partial x} + v' \frac{\partial \bar{\theta}_0}{\partial y} \right) - \frac{\partial^2 \theta'}{\partial y^2} - \eta^2 \frac{\partial^2 \theta'}{\partial x^2} = 0. \quad (57)$$

Interfacial balance equations:

$$\frac{\partial u'}{\partial y} + a' \frac{\partial^2 \bar{u}_0}{\partial y^2} + \eta^2 \frac{\partial v'}{\partial x} = 0 \quad (58)$$

$$WeRe\eta^2 \frac{\partial^2 a'}{\partial x^2} + p' + a' \frac{\partial \bar{p}_0}{\partial y} - 2\eta \frac{\partial v'}{\partial y} = 0 \quad (59)$$

$$v' - \bar{u}_0 \frac{\partial a'}{\partial x} - \frac{\partial a'}{\partial t} + a' \frac{\partial \bar{v}_0}{\partial y} - u' \frac{\partial \bar{a}_0}{\partial x} + \frac{\partial \theta'}{\partial y} = 0. \quad (60)$$

Boundary conditions:

at the wall, $y = 0$

$$u' = 0 \quad (61)$$

$$v' = 0 \quad (62)$$

$$\left. \begin{array}{l} \text{CT: } \theta' = 0 \\ \text{CHF: } \frac{\partial \theta'}{\partial y} = 0 \end{array} \right\} \quad (63)$$

at the interface, $y = a_0(x)$

$$\theta' + a' \frac{\partial \bar{\theta}_0}{\partial y} = 0. \quad (64)$$

The system (54)–(64) differs from the corresponding system of an isothermal flow by the coupling between the thermal and dynamical problems through equation (60), a combination of the interfacial energy and mass balance equations. For an isothermal flow, the corresponding equation reads

$$v' - \bar{u}_0 \frac{\partial a'}{\partial x} - \frac{\partial a'}{\partial t} = 0. \quad (65)$$

The reason why the phase change appears only in the interfacial mass and energy balance equations is

the strong limitation imposed on the Kutateladze number or on the temperature difference scale $\Delta\Theta_{Lr}$.

If, for instance, a value $Ku = 0.1$ and a value $Re = 10$ were chosen, the phase change term in the interfacial momentum equation would have an order of magnitude of 1 (first order), the convection and inertial terms in the liquid of 0.1, and physical properties variation terms of 0.5 (viscosity).

Nevertheless, if convection, inertia and physical properties variation are neglected, the interfacial momentum balance equation of the perturbed flow would read (first order)

$$WeRe\eta^2 \frac{\partial^2 a'}{\partial x^2} + p' + a' \frac{\partial \bar{p}_0}{\partial y} - 2\eta \frac{\partial v'}{\partial y} + \frac{2\eta^2 Re}{\gamma} \frac{\partial \bar{\theta}_0}{\partial y} \frac{\partial \theta'}{\partial y} - \frac{\eta^2 Re}{\gamma} p^{*'} + \frac{2M_r \eta^3}{\gamma} \frac{\partial^2 a'}{\partial x^2} \bar{v}_{G0} = 0. \quad (66)$$

Three supplementary terms appear in comparison with equation (59):

The phase change term

$$\frac{\partial \bar{\theta}_0}{\partial y} \cdot \frac{\partial \theta'}{\partial y}$$

and two vapor terms \bar{v}_{G0} and $p^{*'}$

$$[\bar{v}_{G0}(a_0) = -a_0^2 \partial a_0 / \partial x, \text{ given in [10]}].$$

Consequently, if the phase change term has to be taken into account in the momentum balance equation, the term $p^{*'}$ has also to be considered, and can only be calculated after solving the whole vapor perturbed equations set.

5. STABILITY STUDY

5.1. Eigenvalue system

The following perturbations of the stream function, temperature and thickness are introduced in the perturbed flow equations

$$\begin{aligned} \psi(x, y, t) &= \phi(y) \exp \left[\frac{i\alpha}{\eta} (x - ct) \right] \\ \theta'(x, y, t) &= Z(y) \exp \left[\frac{i\alpha}{\eta} (x - ct) \right] \end{aligned} \quad (67)$$

$$a'(x, t) = S \exp \left[\frac{i\alpha}{\eta} (x - ct) \right].$$

After some classical rearrangements, the following eigenvalue system is obtained (index 0 of base first-order system is omitted):

$$\begin{aligned} \phi^{IV}(y) - 2\alpha^2 \phi''(y) + \alpha^4 \phi(y) &= i\alpha Re \\ &\times \left\{ (\bar{u} - c) [\phi''(y) - \alpha^2 \phi(y)] - \frac{\partial^2 \bar{u}}{\partial y^2} \phi(y) \right\} \end{aligned} \quad (68)$$

$$Z''(y) - \alpha^2 Z(y) = i\alpha Pe \left[(\bar{u} - c) Z(y) - \frac{\partial \bar{\theta}}{\partial y} \phi(y) \right] \quad (69)$$

$$\phi(0) = 0 \quad (70)$$

$$\phi'(0) = 0 \quad (71)$$

$$\left. \begin{aligned} \text{CT: } Z(0) &= 0 \\ \text{CHF: } Z'(0) &= 0 \end{aligned} \right\} \quad (72)$$

$$\phi'''(a) + [Rei\alpha(c - \bar{u}) - 3\alpha^2]\phi'(a) - i\alpha(\cotg \Omega + \alpha^2 WeRe)S = 0 \quad (73)$$

$$\phi''(a) + \alpha^2\phi(a) - S = 0 \quad (74)$$

$$Z(a) + \frac{\partial \bar{\theta}}{\partial y} S = 0 \quad (75)$$

$$rS = \phi(a) + \frac{\eta}{i\alpha} \frac{\partial a}{\partial x} \phi'(a) - \frac{\eta}{i\alpha} Z'(a) \quad (76)$$

with

$$r \triangleq c - \bar{u}(a) + \frac{\eta}{i\alpha} \frac{\partial \bar{v}}{\partial y} \Big|_a \quad (77)$$

The four interfacial conditions (73)–(76) can be reduced to three by eliminating the constant S :

$$\phi'''(a) + [Rei\alpha(c - \bar{u}) - 3\alpha^2]\phi'(a) - i\alpha \frac{(\cotg \Omega + \alpha^2 WeRe)}{c - \bar{u}} \phi(a) = 0 \quad (78)$$

$$\phi''(a) + \left(\alpha^2 - \frac{1}{r}\right)\phi(a) - \frac{\eta}{i\alpha r} \frac{\partial \bar{a}}{\partial x} \phi'(a) + \frac{\eta}{i\alpha r} Z'(a) = 0 \quad (79)$$

$$Z(a) + \frac{\eta}{i\alpha r} \frac{\partial \bar{a}}{\partial x} \frac{\partial \bar{\theta}}{\partial y} \phi'(a) + \frac{1}{r} \frac{\partial \bar{\theta}}{\partial y} \phi(a) - \frac{\eta}{i\alpha r} \frac{\partial \bar{\theta}}{\partial y} Z'(a) = 0. \quad (80)$$

5.2. Resolution methods

Three methods are considered: a calculation by expansion in power series of α , and two other methods, valid for each value of the wave number. Details of the calculations concerning these methods can be found in a report by Spindler [18].

Expansion in power series of α . An approximate solution of the eigenvalue system (68–76) is obtained for low values of α , developing the functions ϕ and Z as

$$\begin{aligned} \phi(y) &= \phi_0(y) + \alpha\phi_1(y) + O(\alpha^2) \\ Z(y) &= Z_0(y) + \alpha Z_1(y) + O(\alpha^2). \end{aligned} \quad (81)$$

The functions are normalized by setting

$$S = 1. \quad (82)$$

The solutions of the first- and second-order systems can be found in Appendix 2. In order to calculate the complex wave velocity c , an expansion of c is not used, because a better value is obtained from equation (76) where ϕ , ϕ' and Z' are replaced by their expansion. The following results are obtained

$$\text{CT: } C_r = \left[a^2 + \frac{\alpha^2 Re^2 a^{10}}{64} + (\cotg \Omega + \alpha^2 WeRe) \right. \quad (83)$$

$$\left. \times \left(\frac{eKu}{2aPrRe} + \frac{5\alpha^2 Rea^7}{72} \right) \right] \Big/ \left(1 + \frac{25\alpha^2 Re^2 a^8}{576} \right) \quad (83)$$

$$\alpha C_i = \left[\frac{3eKu}{a^2 PrRe} + \frac{2\alpha^2 Rea^6}{15} - \frac{a^3}{3} \alpha^2 \right. \quad (84)$$

$$\left. \times (\cotg \Omega + \alpha^2 WeRe) \right] \Big/ \left(1 + \frac{25\alpha^2 Re^2 a^8}{576} \right) \quad (84)$$

$$\text{CHF: } C_r = \left[a^2 + \frac{\alpha^2 Re^2 a^{10}}{64} + (\cotg \Omega + \alpha^2 WeRe) \right. \quad (85)$$

$$\left. \times \left(\frac{eKu}{2PrRe} + \frac{5\alpha^2 Rea^7}{72} \right) \right] \Big/ \left(1 + \frac{25\alpha^2 Re^2 a^8}{576} \right) \quad (85)$$

$$\alpha C_i = \left[\frac{2eKu}{aPrRe} + \frac{2\alpha^2 Rea^6}{15} - \frac{a^3}{3} \alpha^2 \right. \quad (86)$$

$$\left. \times (\cotg \Omega + \alpha^2 WeRe) \right] \Big/ \left(1 + \frac{25\alpha^2 Re^2 a^8}{576} \right) \quad (86)$$

Method of Anshus and Goren. The principle of the method of Anshus and Goren [19] is to replace the base flow velocity profile $\bar{u}(y)$ in the equations (68) and (69) by the interfacial value $\bar{u}(a)$. The equations become then equations with constant coefficients with a solution of the form

$$\begin{aligned} \phi(y) &= A \sin \beta_1 y + B \cos \beta_1 y + C \sin \beta_2 y + D \cos \beta_2 y \\ Z(y) &= E \sin \beta_1 y + F \cos \beta_1 y + G \sin \beta_2 y + H \cos \beta_2 y \\ &\quad + I \sin \beta_3 y + J \cos \beta_3 y. \end{aligned} \quad (87)$$

The six boundary conditions (70)–(72) and (78)–(80) become then a homogeneous linear system of six equations with six unknown variables: A, B, C, D, I and J . There exists a non-trivial solution if the determinant is equal to zero. A relation $f(\alpha, C, Re, Pr, Ku, We, \Omega) = 0$ is then obtained.

The wall conditions (70) and (71) lead very simply to a reduction in the determinant order from 6 to 4. A numerical resolution is performed in order to calculate the complex celerity in terms of the wave number and the flow parameters. The calculation is initialized ($\alpha = 0$) with the results obtained by the expansion method [equations (83)–(86)].

Method of Solésio. The method of quadrature by differentiation developed by Solésio [20] consists in approximating the integral of a function by a development which involves the values of the successive derivatives at the two end points

$$\int_0^a f(y) dy \approx \frac{1}{C_0^n} \sum_{k=0}^{n-1} C_{k+1}^n a^{k+1} \times [f^{(k)}(0) + (-1)^k f^{(k)}(a)] \quad (88)$$

with

$$C_k^n = \frac{(2n - k)!}{(n - k)!k!}.$$

The two functions ϕ and Z are expanded by power series in y at end point $y = 0$ and in $(y - a)$ at end point $y = a$;

$$\text{at } y = 0: \phi(y) = \sum_{j=0}^P b_j y^j \tag{89}$$

$$Z(y) = \sum_{j=0}^Q d_j y^j$$

$$\text{at } y = a: \phi(y) = \sum_{j=0}^P e_j (y - a)^j \tag{90}$$

$$Z(y) = \sum_{j=0}^Q f_j (y - a)^j$$

Substituting equations (89) and (90) in the eigenvalue system, one obtains two linear homogeneous system of $P + Q - 1$ equations, the unknowns of which are the $P + Q + 2$ coefficients b_j and d_j on one hand, e_j and f_j on the other hand. All the coefficients are then expressed in terms of six of them. A further system of six linear homogeneous equations is obtained applying the quadrature formula with $n = P - 3$ to the functions ϕ', ϕ'', ϕ''' and ϕ^{IV} and with $n = Q - 1$ to the functions Z' and Z'' .

In order to have a non-trivial solution it is required that the determinant (order 6) of the equations set be equal to zero. This gives the relation $f(\alpha, C, Re, Pr, Ku, We, \Omega) = 0$. A numerical resolution is made, as it was done also in the method of Anshus and Goren; sufficient precision is obtained by taking $P = 6$ and $Q = 4$.

Comparison. The results obtained from the calculation by expansion in power series are only valid for low values of the wave number ($\alpha \leq 10^{-2}$). The deviation from the other methods increases when the Reynolds number becomes larger. Nevertheless, the neutral stability curves ($\alpha_c = 0$) are well predicted with the method of expansion in α .

The two other methods give almost the same results. The method of Anshus and Goren was used (determinant of order 4), because it requires a calculation time twice shorter than the method of Solésio (determinant of order 6). Nevertheless the method of Solésio gives a validation of the method of Anshus and Goren.

5.3. Stability of the flow with a preexisting film

First the study of a perturbation initiated at the initial abscissa ($x_0 = 0$) leads to the study of the influence of the phase change itself. A second step consists in studying the stability of another perturbation, initiated at an abscissa different from zero, a case for which the thickness variation influence can be investigated.

Phase change influence. The calculations of the growth factor α_c made at the initial abscissa lead to the conclusion that the evaporation has a destabilizing effect (increase of α_c), whereas the condensation has a

stabilizing one (decrease of α_c) (Fig. 3). The phase change influence increases when the Kutateladze number becomes larger, and decreases when the wave number, Reynolds and Prandtl numbers become larger.

In case of condensation ($e \triangleq -1$), there exists a critical Reynolds number below which the flow is always stable. This critical Reynolds number appears clearly on the neutral stability curves ($\alpha_c = 0$) (Fig. 4). Its value calculated from equations (84) and (86) for a vertical flow reads

$$\text{CT: } Re_c = a^{-3} \left[225 \frac{Ku}{Pr} \frac{\sigma}{\rho} (gv^4)^{-1/3} \right]^{3/11} \tag{91}$$

$$\text{CHF: } Re_c = a^{-3} \left[150 \frac{C_p e q_w \sigma}{Pr \lambda L \rho} (gv)^{-2/3} \right]^{3/10}$$

For a vertical water flow at 100°C (CT, $Ku = 10^{-2}$, $x = 0$), one obtains $Re_c = 14.8$. For a non-vertical flow, the value of Re_c corresponding to a perturbation born at $x = 0$ is quite the same as for an isothermal flow, for which the calculation by expansion in power series [21] gives

$$Re_c = \frac{5}{2} \cotg \Omega. \tag{92}$$

The neutral stability curves corresponding to evaporation and condensation cannot be distinguished above $Re = 30$ for a vertical flow, and above Re_c for a non-vertical flow.

As already explained, phase change terms only appear in the combination of the interfacial mass and energy balance equations. The phase change effect is then purely kinetic and not at all dynamic. The physical interpretation is the following. Examine for instance the case of condensation. At the trough of a wave, the film is a little thinner than at the crest. Then the rate of phase change is a little larger at the trough

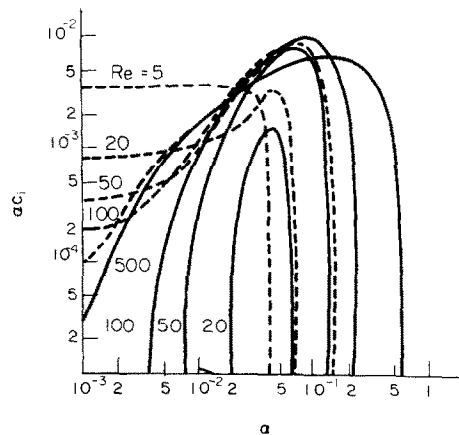


FIG. 3. Flow with a pre-existing film. Temporal growth factor versus wave number. Vertical flow of water, CT, $x = 0$, $Ku = 10^{-2}$, $Pr = 1.7$. — condensation; -- evaporation.

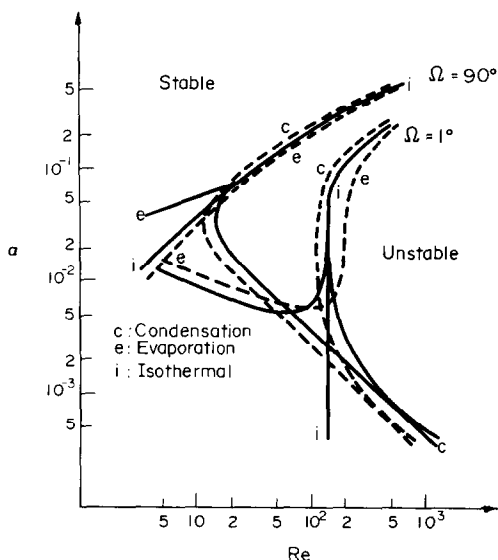


FIG. 4. Flow with a pre-existing film. Neutral stability curves. Water flow, CT, $Ku = 10^{-2}$. — $x = 0$; --- $x = 0.1$.

and consequently an excess in condensing liquid appears compared to what happens at the crest. This leads to a smaller wave amplitude (stabilization).

Comparison can only be made with the study of Kocamustafaogullari [3], where the phase change effect is taken into account in the interfacial mass balance, but the thickness variation is neglected. The neutral stability curve obtained by Kocamustafaogullari is given by the following equation (CHF, vertical flow, $Ku \ll 1$):

$$\eta\gamma + \frac{Re\alpha^2}{9} - \frac{WeRe\alpha^4}{3} = 0. \quad (93)$$

It has to be compared with the following equation established from equation (86) (CHF, vertical flow, $x = 0$):

$$2\eta\gamma + \frac{2Re\alpha^2}{15} - \frac{WeRe\alpha^2}{3} = 0. \quad (94)$$

Figure 5 shows that the two curves are not very different. As a result $Re_c = 9.9$ for Kocamustafaogullari and 10.9 for our study (vertical flow of water at 100°C , $q_w = 4 \times 10^4 \text{ W m}^{-2}$). A difference between the first terms of the two equations (93) and (94) appears because Kocamustafaogullari does not take into account the film thickness variation [terms with $\partial a/\partial x$ and $\partial v/\partial y$ in equation (60)]. The difference between the second terms can only be explained by the different set of equations used: local equations (our study) or equations averaged over the film thickness (Kocamustafaogullari).

Thickness variation influence. The study of perturbations born at different distances shows that a thickness decrease (evaporation) has a stabilizing

effect whereas a thickness increase (condensation) has a destabilizing effect. For a flow with a Reynolds number of 50, at the abscissa $x = 0.01$ ($X = 6.4 \text{ mm}$) the kinetic effect is still greater than the thickness variation effect until about the maximum growth factor, then becomes lower. On the other hand, at the abscissa $x = 0.1$ ($X = 6.4 \text{ cm}$), the thickness variation effect is large and becomes greater than the phase change effect as soon as the wave number becomes larger than about 10^{-2} (Fig. 6).

For a Reynolds number of 250, the thickness variation remains low at $x = 0.1$ ($X = 55 \text{ cm}$) (Fig. 7). And, as the waves certainly start at a distance lower than 50 cm, one concludes that the thickness variation is of low influence on the linear stability at high Reynolds number.

5.4. Stability of a condensation film flow

As in case of a flow with pre-existing film, the condensation has a stabilizing effect, which is important only for low wave number. It is found that when the abscissa or the Reynolds number [equation (46)] increases the condensation effect reduces: the thickness increase destabilizes the flow (Fig. 8). There exists a region near the origin ($\alpha = 0$) where the flow is stable ($\alpha_c < 0$). Moreover the flow is always stable for perturbation born at an abscissa lower than a critical distance X_c , for any wave number. Calculated from equations (84) and (86) for a vertical flow, the critical distance is

$$\text{CT: } X_c = \frac{1}{4} \left[15^8 \frac{Pr^7}{Ku^7} \left(\frac{\sigma}{\rho} \right)^4 \frac{v^2}{g^5} \right]^{1/11} \quad (95)$$

$$\text{CHF: } X_c = \frac{1}{3} \left[150^3 \left(\frac{\sigma}{\rho} \right)^3 \frac{L^7 \mu^7}{g^2 v^2 q^7} \right]^{1/10}.$$

For water at 100°C (CT, $\Delta\Theta = 5.5 \text{ K}$) one obtains $X_c = 3.18 \text{ cm}$ ($Re_c = 5.24$). The same value is given by the Anshus and Goren method.

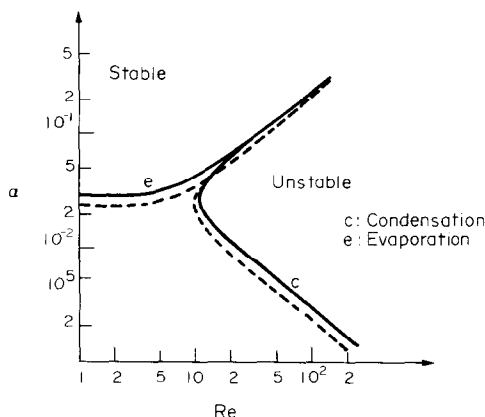


FIG. 5. Flow with a pre-existing film. Neutral stability curves. Vertical flow of water, CHF, $q_w = 4 \times 10^4 \text{ W m}^{-2}$, $x = 0$. — Spindler [equation (94)]; --- Kocamustafaogullari [equation (93)].

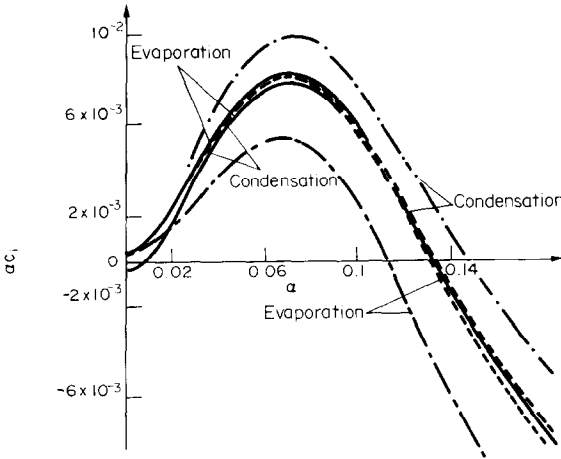


FIG. 6. Flow with a pre-existing film. Temporal growth factor vs wave number. Vertical flow of water, CT, $Ku = 10^{-2}$, $Re = 50$. — $x = 0$; - - $x = 0.01$ ($X = 6.4$ mm); - · - $x = 0.1$ ($x = 6.4$ cm).

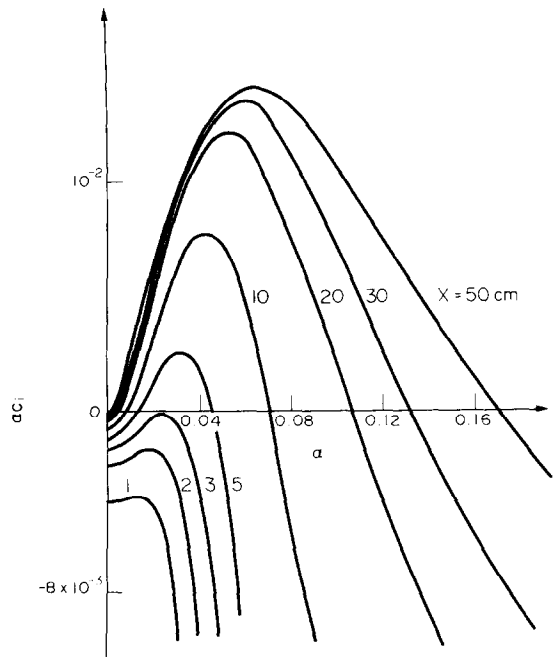


FIG. 8. Condensation film flow. Temporal growth factor vs wave number. Vertical flow of water, CT, $Ku = 10^{-2}$.

These results can only be compared with the study of Unsal and Thomas [8], who calculate the growth factor by means of an expansion in power series of α . They obtain (CT, vertical flow, $Ku \ll 1$),

$$\alpha_c_i = \left(-\frac{Ku}{2PrRe} + \frac{16\alpha^2 Re}{15} - \frac{2\sqrt{2}}{3} WeRe\alpha^4 \right) \left(1 + \frac{25\alpha^2 Re^2}{36} \right), \quad (96)$$

which has to be compared with equation (84) where $\Omega = \pi/2$ and $a = \sqrt{2}$ [$x = 1$, equation (43)].

A difference appears only in the first term, corresponding to the interfacial mass balance equation, which Unsal and Thomas write as:

$$v' - \bar{u} \frac{\partial a'}{\partial x} - \frac{\partial a'}{\partial t} + \frac{\partial \theta'}{\partial y} = 0 \quad (97)$$

instead of equation 60 (Fig. 9).

5.5. Remarks

Wall thermal condition. The same effect of the phase change as well as of the thickness variation on the flow stability is found whatever the thermal condition is: constant temperature or constant heat flux. The difference is sensitive for very low wave numbers only. The ratio of the two (CT to CHF) growth factor is equal to 2/3 [equations (84) and (86)].

Perturbation velocity. The phase change influence on the perturbation velocity is very low. On the other hand the thickness variation is important (Fig. 10).

Temporal stability-spatial stability. The results are obtained with the temporal formulation [equation

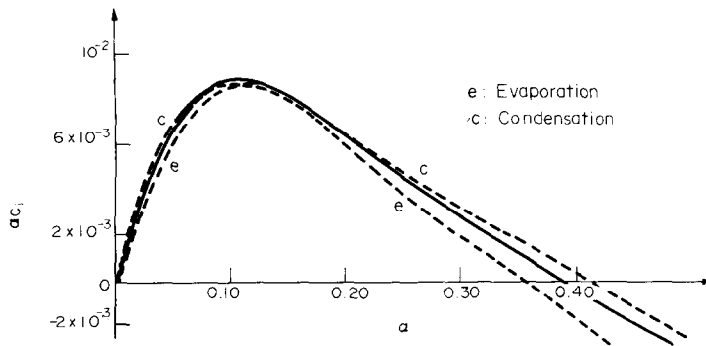


FIG. 7. Flow with a pre-existing film. Temporal growth factor vs wave number. Vertical flow of water, CT, $Ku = 10^{-2}$, $Re = 250$. — $x = 0$; - - $x = 0.1$ ($X = 55$ cm).

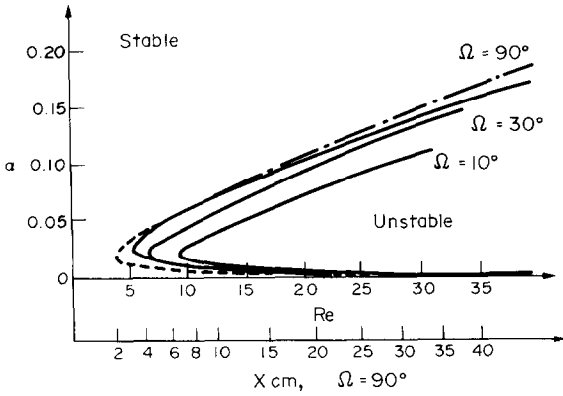


FIG. 9. Condensation film flow. Neutral stability curves. Water flow, CT, $Ku = 10^{-2}$. — Anshus and Goren; -- Unsal and Thomas [from equation (96)]; - · - · expansion in α [from equations (96) and (43)].

(53)] because it is much more convenient to make the calculation with α as a real number. Nevertheless spatial stability can also be considered

$$f'(x, y, t) = F(y) \exp \left[\frac{i}{\eta} (\alpha x - \omega t) \right] \quad (98)$$

(α complex and ω real).

The same influence of both phase change and thickness variation is found.

6. CONCLUSION

The order of magnitude analysis of the dimensionless equations terms leads to a proper evaluation of the influence of the different terms such as phase change

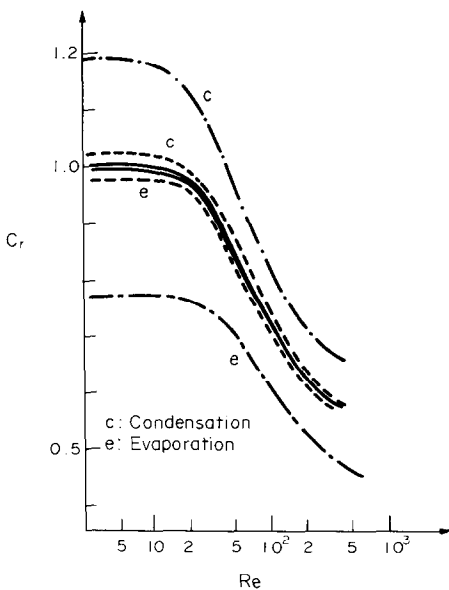


FIG. 10. Flow with a pre-existing film. Velocity of the most amplified wave (α_c , maximum). Vertical flow of water, CT, $Ku = 10^{-2}$. — $x = 0$; -- $x = 0.01$; - · - · $x = 0.1$.

terms, vapor terms and physical properties variation terms.

For flows with a pre-existing film as well as for condensation film flows, a phase change term has to be taken into account in the interfacial mass balance equation. Moreover that phase change term only is of important influence in case of limited wall heat flux or wall to interface temperature difference ($Ku \ll 1$).

On the other hand, in case of large wall heat flux or wall to interface temperature difference a phase change term in the interfacial momentum equation has to be considered, along with some supplementary terms. Among these a term corresponding to the vapor flow has to be taken into account.

For low Kutateladze number, condensation has a stabilizing effect whereas evaporation has a destabilizing one.

The calculated film thickness variation caused by phase change has an opposite effect to the phase change one.

Both phase change and thickness variations influence on the stability are particularly sensitive in case of low Reynolds numbers ($Re < 100$).

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APPENDIX

1. Value of the scales

Transversal length in the liquid
 Transversal length in the vapor
 Longitudinal length in the vapor
 Ratio of the liquid length scales
 Ratio of the vapor length scales
 Longitudinal velocity in the liquid and in the vapor
 Transversal velocity in the phase k
 Vapor to liquid longitudinal velocity difference at the interface
 Vapor to liquid transversal velocity difference at the interface
 Time in the phase k
 Pressure difference in the liquid
 Pressure difference in the vapor
 Modified pressure difference in the vapor
 Liquid to vapor interfacial pressure difference
 Pressure
 Temperature difference in the liquid
 Temperature difference in the vapor
 Temperature

$$Y_{lr} = [\Delta\Theta_{lr}\lambda_{lr}\mu_{lr}X_{lr}/L\rho_{lr}^2g \sin \Omega]^{\frac{1}{4}}$$

$$Y_{gr} = M_r Pr_L Y_{lr} / Ku$$

$$X_{gr} = \gamma_r M_r Pr_L X_{lr} / Ku$$

$$\eta_L = Ku / Pr_L Re_L$$

$$\eta_G = \eta_L / \gamma_r$$

$$U_{gr} = U_{lr} = \rho_{lr} g \sin \Omega Y_{lr}^2 / \mu_{lr}$$

$$V_{kr} = \eta_k U_{kr}$$

$$\Delta U_r = \eta_L^2 U_{lr} / \gamma_r$$

$$\Delta V_r = \eta_L U_{lr} / \gamma_r$$

$$T_{kr} = X_{kr} / U_{kr}$$

$$\Delta P_{lr} = \rho_{lr} U_{lr}^2 / Re_L$$

$$\Delta P_{gr} = \gamma_r^2 Pr_L^2 M_r \rho_{lr} U_{lr}^2 / Ku^2$$

$$\Delta P_r^* = \eta_L^2 \rho_{lr} U_{lr}^2 / \gamma_r$$

$$\Delta P_{ir} = \eta_L^2 \rho_{lr} U_{lr}^2 / \gamma_r$$

$$P_{lr} = P_{gr} = P_r^* = P_G [X = 0, Y = A(X, T)]$$

$$\left\{ \begin{array}{l} \text{CT: } \Delta\Theta_{lr} \triangleq e(\Theta_w - \Theta_{lr}) \\ \text{CHF: } \Delta\Theta_{lr} \triangleq eq_w Y_{lr} / \lambda_{lr} \end{array} \right.$$

$$\Delta\Theta_{gr} = (d\Theta_S/dP_S)\gamma_r^2 M_r Pr_L^2 \rho_{lr} U_{lr}^2 / Ku^2 Re_L^2$$

$$\Theta_{lr} = \Theta_{gr} = \Theta_S(X = 0).$$

2. Expansion in power series of α : value of ϕ_0 , Z_0 , ϕ_1 and Z_1

$$\phi_0(y) = \frac{y^2}{2}$$

$$\text{CT: } Z_0(y) = \frac{e}{a^2} y$$

$$\text{CHF: } Z_0(y) = e$$

$$\phi_1(y) = iRe \left[\frac{ay^5}{120} - \frac{cy^4}{24} + \frac{a^2 y^2}{4} \left(c - \frac{a^2}{3} \right) \right] + i(\cotg \Omega + \alpha^2 WeRe) \left(\frac{y^3}{6} - \frac{ay^2}{2} \right)$$

$$\text{CT: } Z_1(y) = eiPe \left[-\frac{y^5}{40a^2} + \frac{y^4}{8a} - \frac{cy^3}{6a^2} - \frac{y}{2} \left(\frac{a^2}{5} - \frac{c}{3} \right) \right]$$

$$\text{CHF: } Z_1(y) = eiPe \left(\frac{ay^3}{6} - \frac{cy^2}{2} - \frac{a^4}{6} + \frac{ca^2}{2} \right).$$

STABILITE LINEAIRE DES FILMS LIQUIDES AVEC CHANGEMENT DE PHASE A L'INTERFACE

Résumé—On étudie l'influence du changement de phase à l'interface sur la stabilité linéaire de l'écoulement de films liquides sur une paroi plane. Elle n'apparaît que par l'intermédiaire de l'équation de bilan de masse à l'interface. L'évaporation déstabilise l'écoulement alors que la condensation le stabilise. Il est tenu compte de la variation d'épaisseur due au changement de phase, qui a un effet opposé à celui du changement de phase lui-même.

LINEARE STABILITÄT VON FLÜSSIGKEITSFILMEN MIT PHASENÄNDERUNG AN DER GRENZFLÄCHE

Zusammenfassung—Untersucht wird der Einfluß der Phasenänderung an der Grenzfläche auf die Stabilität einer Flüssigkeitsfilm-Strömung über eine ebene Fläche. Er ist nur in der Massenbilanz für die Grenzschicht von Bedeutung. Verdunstung hat einen destabilisierenden, Kondensation dagegen einen stabilisierenden Einfluß. Die durch den Phasenübergang bedingte Änderung der Filmdicke wird berücksichtigt. Ihre Wirkung auf die Stabilität ist jener der Phasenänderung entgegengesetzt.

ЛИНЕЙНАЯ УСТОЙЧИВОСТЬ ЖИДКИХ ПЛЕНОК ПРИ ФАЗОВОМ ПРЕВРАЩЕНИИ НА ГРАНИЦЕ РАЗДЕЛА

Аннотация — Исследуется влияние фазового превращения на границе раздела на устойчивость обтекания пластины жидкой пленкой. Это влияние следует учитывать только в уравнении межфазного баланса массы. Испарение дестабилизирует течение, в то время как конденсация оказывает стабилизирующее воздействие. Учитывается изменение толщины пленки, обусловленное фазовым превращением, влияние которого на устойчивость течения противоположно.